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Corrigendum to “Reconciling the Rawlsian and the utilitarian approaches to the maximization of social welfare” [Economics Letters 122 (2014) 439–444]

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Subsection 3.1. (pp. 441–442) of the originally published article analyzes the optimal choice of a Rawlsian social planner (RSP). The originally published subsection did not cover all possible cases, and the proof that a RSP will choose to equalize incomes is incomplete. The text that follows replaces that subsection.

3.1. *The maximization problem of a Rawlsian social planner*

The maximization problem of a RSP is

$$\max_{\Omega(a_1,\dots,a_n;\lambda)} SWF_R(x_1,\dots,x_n) = \max_{\Omega(a_1,\dots,a_n;\lambda)} \left\{\min\{u_1(x_1,\dots,x_n),\dots,u_n(x_1,\dots,x_n)\}\right\}. \tag{3}$$

It is easy to see that for every $k\in\{1,\dots,n-1\}$ we have that

$u_i(x_1,\dots,x_k,x_{k+1},\dots,x_n)=u_i(x_1,\dots,x_{k+1},x_k,\dots,x_n)$

for $i\in\{1,\dots,n\}\setminus\{k,k+1\}$, and that

$u_k(x_1,\dots,x_k,x_{k+1},\dots,x_n)=u_{k+1}(x_1,\dots,x_{k+1},x_k,\dots,x_n)$.

Therefore, if $x_1\le\dots\le x_n$, then the monotonicity of the f function and the definition of the RI function imply that $u_1(x_1,\dots,x_n)\le u_2(x_1,\dots,x_n)\le\dots\le u_n(x_1,\dots,x_n)$. Thus, for any k such that $y_k=\min\{y_1,\dots,y_n\}$, we have that $SWF_R(y_1,\dots,y_n)=u_k(y_1,\dots,y_n)$.

Denoting by $(x_1^{R^*},\dots,x_n^{R^*})$ the optimal post-transfer distribution of incomes of a RSP, we have that

$$\max_{\Omega(a_1,\dots,a_n;\lambda)} SWF_R(x_1,\dots,x_n)=u_1(x_1^{R^*},\dots,x_n^{R^*}),$$

where $x_1^{R^*}=\dots=x_n^{R^*}$. We prove this claim by contradiction. To do that, we assume that $(x_1^{R^*},\dots,x_n^{R^*})\in\Omega(a_1,\dots,a_n;\lambda)$ is such that $\underline{x}=\min\{x_1^{R^*},\dots,x_n^{R^*}\}<\max\{x_1^{R^*},\dots,x_n^{R^*}\}$, and we show that there exists $(y_1,\dots,y_n)\in\Omega(a_1,\dots,a_n;\lambda)$ such that $SWF_R(y_1,\dots,y_n) > SWF_R(x_1^{R^*},\dots,x_n^{R^*})$. Therefore, $(x_1^{R^*},\dots,x_n^{R^*})$ cannot be a maximum.

Let $I=\{i\in\{1,\dots,n\}:x_i^{R^*}=\underline{x}\wedge x_i^{R^*}\ge a_i\}$, $J=\{i\in\{1,\dots,n\}:x_i^{R^*}=\underline{x} \wedge x_i^{R^*}<a_i\}$, $\bar{x}=\min\{x_i:i\notin I\cup J\}$, $k=\min\{i\in\{1,\dots,n\}:x_i^{R^*}=\bar{x}\}$, $K=I\cup J \cup\{k\}$, and $h=|I\cup J|$, where the notation $|A|$ stands for the cardinality of the set A. Obviously, from the characteristics of the point $(x_1^{R^*},\dots,x_n^{R^*})$, it follows that $I\cup J\neq\varnothing$ and that $h\ge 1$. Let δ be such that $0<\delta<\min\left\{\lambda(\bar{x}-\underline{x})/2, \min\limits_{i\in K:a_i\neq x_i^{R^*}}\left\{\left|a_i-x_i^{R^*}\right|\right\}\right\}$. We now define the coordinates of the point $(y_1,\dots,y_n)$ as

$$y_i=\begin{cases} x_i^{R^*}+\delta/h & \text{for } i\in I\cup J,\\ x_i^{R^*}-\delta_k & \text{for } i=k,\\ x_i^{R^*} & \text{for } i\in\{1,\dots,n\}\setminus K,\end{cases}$$

where $\delta_k=\delta(|I|+\lambda|J|)/(\lambda h)$ if $x_k^{R^*}\le a_k$, and $\delta_k=\delta(|I|+\lambda|J|)/h$ otherwise. It is easy to verify that $(y_1,\dots,y_n)\in\Omega(a_1,\dots,a_n;\lambda)$.

Because the f function is an increasing function, and because a smaller difference between incomes implies a smaller value of the index of low relative income, it follows that for any $i\in I\cup J$

$$\begin{aligned} &SWF_R(y_1,\dots,y_n)-SWF_R(x_1^{R^*},\dots,x_n^{R^*})\\ &\quad=u_i(y_1,\dots,y_n)-u_i(x_1^{R^*},\dots,x_n^{R^*})\\ &\quad=(1-\beta)\left[f(x_i^{R^*}+\delta/h)-f(x_i^{R^*})\right]\\ &\qquad-\beta\left[RI(x_i^{R^*}+\delta/h;y_1,\dots,y_n)-RI(x_i^{R^*};x_1^{R^*},\dots,x_n^{R^*})\right]>0\end{aligned}$$

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for any $\beta \in [0,1)$ and $0 < \lambda \leq 1$. Therefore, $SWF_R(y_1, \ldots, y_n) > SWF_R(x_1^{R^*}, \ldots, x_n^{R^*})$, which contradicts the fact that SWF_R attains a global maximum at $(x_1^{R^*}, \ldots, x_n^{R^*})$. Thus, the solution of the problem of a Rawlsian social planner, (3), has to be a transfer such that the post-transfer incomes are all equal. This completes the proof by contradiction.

It is worth noting that the solution of (3) is unique. To show this, we assume that $a_1 < a_n$, and we let

$$g(x) = \frac{\sum_{i=1}^{n} \max\{x - a_i, 0\}}{\sum_{i=1}^{n} \max\{a_i - x, 0\}}$$

for $x \in [a_1, a_n)$. Then, as a ratio of a continuous, strictly increasing function and a continuous, strictly decreasing and positive function, g is continuous and strictly increasing, and $g(a_1) = 0$, $\lim_{x \to a_n} g(x) = \infty$. Therefore, there exists a unique $x^{R^*} \in (a_1, a_n)$ such that $g(x^{R^*}) = \lambda$, which is the solution of $\lambda \sum_{i=1}^{n} \max\{a_i - x, 0\} = \sum_{i=1}^{n} \max\{x - a_i, 0\}$, and we have that $x^{R^*} = x_1^{R^*} = \ldots = x_n^{R^*}$.

Concluding this subsection, we note that the distribution chosen by a RSP entails equality of incomes even when $\beta = 0$, namely, even if individuals' concern at having low relative income is excluded from the RSP's social welfare function.